Impact protective performance and theoretical analysis of polyvinyl chloride/thermoplastic polyurethane foam composite structure via finite element simulation

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SUPPLEMENT FIGURES:

Figure S1. Custom-developed impact tester device and Rebound height monitoring

Figure S2. a) uniaxial compression; b) equibiaxial compression

The principal tensile and nominal stress in the loading direction are

\[
\lambda_1 = 1 + \varepsilon_1 = 1 - \frac{u_1}{L_0} \quad \lambda_2 = 1 + \varepsilon_2 = 1 - \frac{u_2}{L_0} \quad (S1)
\]

\[
P_1 = \frac{F_1}{L_0} \quad P_2 = \frac{F_2}{L_0} \quad (S2)
\]

The corresponding equal biaxial nominal stress and principal tensile are obtained by conversion

\[
P_2 = \frac{F_2}{L_0} = \frac{F}{\sqrt{2}L_0^2} \quad \lambda_2 = 1 - \frac{u_2}{L_0} = 1 - \frac{u}{\sqrt{2}L_0} \quad (S3)
\]
Figure S3. Photos under different compressive strains.

Figure S4. When N= 1,2,3, the stress-strain curve is compared with the stress-strain curve measured by the experiment.

Figure S5. Iterative step flow of Rayleigh damping parameters
**Figure S6.** Von Mises stress distribution on a PTFE+TPU foam at the maximum impact state in the free-falling sphere simulation viewed from (a1) the top, (a2) the bottom, and the cross-section. Von Mises stress distribution on a TPU+TPU foam at the maximum impact state in the free-falling sphere simulation viewed from (b1) the top, (b2) the bottom, and the cross-section.

**Figure S7.** Evolution of von Mises stress during the initial impact process on (a) the PTFE+TPU foam and (b) the TPU+TPU foam.

**SUPPLEMENT EQUATIONS:**

The relation between the second Piola-Kirchhoff stress and strain energy density is as follows

\[ S = 2 \frac{\partial W}{\partial C} \]  \hspace{1cm} (S4)

Where \( C \) is the right Cauchy-Green deformation tensor. The principal true stress can be expressed as a function of the second Piola-Kirchhoff stress, as follows

\[ \sigma = J^{-1}FSF^T \]  \hspace{1cm} (S5)

For uniaxial and equibiaxial deformation, the deformation gradient \([F]\) is
The relationship between principal true stress and the principal stretch rate is given by

$$\sigma_i = J^{-1} \lambda_i \frac{\partial W}{\partial \lambda_i} \quad i = 1, 2, 3$$  \hspace{1cm} (S7)$$

The principal stretch rate for uniaxial deformation of isotropic hyperelastic materials is given by

$$\lambda_1 = \lambda_3, \lambda_2 = \lambda$$  \hspace{1cm} (S8)

$$J = J_e = \lambda^2$$  \hspace{1cm} (S9)$$

The principal true stresses in principal directions 1 and 3 are as follows

$$\sigma_1 = \sigma_3 = 0 = J^{-1} \sum_{k=1}^{n} 2 \mu_k \lambda_k^2 \left( \alpha_k \lambda_k^{\mu_k-1} - \alpha_k J^{\mu_k-1} \frac{\partial J}{\partial \lambda_k} \right)$$  \hspace{1cm} (S10)$$

The stretch rate in the 1 direction and $J$ can be obtained

$$\lambda_1^r = \lambda^{-\beta(1+2\beta)}$$  \hspace{1cm} (S11)

$$J = \lambda^{2/(1+2\beta)}$$  \hspace{1cm} (S12)$$

For each $\beta = \beta_k$, the above equation applies. The relationship between nominal stress $\sigma_20$ and true stress $\sigma_2$ is as follows

$$\sigma_20 = \lambda_1^r \sigma_2$$  \hspace{1cm} (S13)$$

For equibiaxial deformation was shown in Fig.1b, the principal stretch rate for uniaxial deformation of isotropic hyperelastic materials is given by

$$\lambda_1 = \lambda_2 = \lambda, \lambda_3 = J \lambda^{-2}$$  \hspace{1cm} (S14)$$
Just like the reasoning for uniaxial deformation, the true stress of equibiaxial deformation is $\sigma_3 = 0$. The stretch rate in the 3 direction and $J$ can be obtained

$$\lambda_3 = \lambda^{(-2\beta)/(1+2\beta)} \quad (S15)$$

$$J = \lambda^{2/(1+\beta)} \quad (S16)$$

For equibiaxial deformation, the relationship between nominal stress $\sigma_{10}$ and true stress $\sigma_j$ is as follows

$$\sigma_{10} = \lambda_2 \lambda_3 \sigma_1 \quad (S17)$$

The conservation of momentum according to the true stress can be expressed by

$$\rho \frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma + f_v \quad (S18)$$

Where $f_v$ is the body force, $\rho$ is the mass density and $u$ is the displacement vector.

The damping matrix $[C]$ is expressed as a linear combination of the mass $[M]$ and stiffness $[K]$ matrices that is

$$C = \alpha_0 M + \alpha_1 K \quad (S19)$$

Where $\alpha_0$ is the mass proportional damping ratio, and $\alpha_1$ represents the stiffness proportional damping ratio.

The uniaxial deformation medium invariants $I_{1u}$ and $I_{2u}$ are expressed through the principal stretch $\lambda$ as

$$I_{1u} = \left( \lambda^2 + \frac{2}{\lambda} \right) \quad (S20)$$

$$I_{2u} = \left( 2\lambda + \frac{1}{\lambda^2} \right) \quad (S21)$$

The equibiaxial deformation medium invariants $I_{1e}$ and $I_{2e}$ are expressed through the principal stretch $\lambda$ as
Nominal stress expressions for uniaxial and equibiaxial deformations for different hyperelastic models were given

**Mooney–Rivlin model:**

\[
\begin{align*}
\sigma_{\text{uniaxial}} &= 2(1-\lambda^{-3}) \left\{ \lambda C_{10} + 2C_{20}\lambda (I_{iu} - 3) + C_{11}\lambda (I_{iu} - 3) + C_{01} + 2C_{02}(I_{iu} - 3) + C_{11} (I_{iu} - 3) \right\} \\
\sigma_{\text{equibiaxial}} &= 2(\lambda - \lambda^{-3}) \left\{ C_{10} + 2C_{20}(I_{ie} - 3) + \lambda^2 C_{01} + 2\lambda^2 C_{02}(I_{ie} - 3) + \lambda^2 C_{11} (I_{ie} - 3) \right\}
\end{align*}
\]  
(S24)

where \( C_{10}, C_{20}, C_{01}, C_{02}, C_{11} \) are material parameters.

**Yeoh model:**

\[
\begin{align*}
\sigma_{\text{uniaxial}} &= 2(\lambda - \lambda^{-3}) \sum_{i=1}^{3} i c_i (I_{iu} - 3)^{i-1} \\
\sigma_{\text{equibiaxial}} &= 2(\lambda - \lambda^{-3}) \sum_{i=1}^{3} i c_i (I_{ie} - 3)^{i-1}
\end{align*}
\]  
(S26)

where \( c_i \) are material parameters.

**Ogden model:**

\[
\begin{align*}
\sigma_{\text{uniaxial}} &= \sum_{i=1}^{N} \mu_i \left( \lambda^{\alpha_i - 1} - \lambda^{-\alpha_i/2 - 1} \right) \\
\sigma_{\text{equibiaxial}} &= \sum_{i=1}^{N} \mu_i \left( \lambda^{\alpha_i - 1} - \lambda^{-2\alpha_i - 1} \right)
\end{align*}
\]  
(S28)

where \( \alpha_i \) and \( \mu_i \) are material parameters.

Table S1: Fitting data parameters of the model.

<table>
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<tr>
<th>( i )</th>
<th>( \mu_i )</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
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For the Storakers model, the parameters must meet the following requirements

\[ \sum_{i=1}^{N} \mu_i > 0, \beta_i > -1/3 \]